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A critical test of stellar evolution and convective core ‘overshooting’ by means of ζ Aurigae systems

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ABSTRACT

Quantitative tests of late stellar evolution are presented by computing models with our evolutionary code to match the exact properties of certain ζ Aurigae eclipsing binaries and related non-eclipsing systems. Those binaries have a late-type giant or supergiant primary and their orbital inclination is well determined from either eclipses or speckle orbits. They provide the only direct measurements of masses for such evolved stars, together with other well-determined physical parameters.

In the computations all effects of enhanced mixing beyond the convective cores during central hydrogen burning stages, e.g., core overshooting and any rotationally induced meridional mixing, are represented by a simple overshooting prescription. Its single parameter can be constrained to within 25 per cent of its value and leads to an overshooting length l_{ov} of $\sim 0.24 H_p$ (pressure scaleheights) for $2.5 M_\odot$, slightly increasing to $\sim 0.32 H_p$ for $6.5 M_\odot$. Those values are required by our code to reproduce the well-determined luminosities of the giants in or at the end of their blue loop. This new method provides the currently most sensitive test of the overshooting issue.

Key words: binaries: eclipsing – stars: evolution – stars: late-type.

1 INTRODUCTION

‘We see that details, which have originated from different regions and from earlier phases when the effects were scarcely recognizable, can now pop up and modify the evolution appreciably. The present phase is a sort of magnifying glass, revealing relentlessly the faults of calculations of earlier phases.’

That is how Kippenhahn & Weigert (1991, p. 305) very nicely pointed out the difficulties encountered in late stellar evolution, referring to central helium burning (‘blue loop’) giants in particular.

For the same reason, a comparison of models from late stellar evolution with real late-type giants should provide the most critical test of stellar evolutionary codes. However, until recently, that approach was hampered by a lack of suitable objects with well enough determined basic parameters, i.e., mass and absolute luminosity. We will return to that point below.

Quantitative tests of stellar evolutionary codes are still vital for the reliability of the models as well as for the development of the codes. For recent reviews and papers on the full complexity of current problems in stellar evolution see, e.g., Vandenberg (1991), Chiosi (1992) and ‘Stellar

Evolution: what should be done’ (Noels et al. 1996). However, the Achilles’ heel of modern evolutionary codes is (still) the very simple representation of convection, mostly by means of the mixing-length theory – compare, e.g., Alongi et al. 1993, Bressan et al. 1993, Schaller et al. 1992 and Schaerer et al. 1993.

A well-known example is the need to adjust the value of α , the mixing length in pressure scaleheights, for the outer convective layers of the cooler stars. In practice, the α value is chosen to make the solar model exactly match the solar effective temperature. Canuto & Mazzitelli (1991, 1992) have tried a parameter-free approach, but it depends on some implicit (though plausible) a priori assumptions and their model fails to match the real Sun precisely.

Massive main-sequence stars have convective cores instead of outer convective layers. In such cases, the mixing length – on which the ratio of convective flux (through heat transport) to radiative flux F_{conv}/F_{rad} and the actual temperature gradient ∇ depend – has no impact on the structure of the models: the efficiency is so large that in practice an adiabatic temperature gradient is achieved in convective cores in any case (whereas in general $\nabla_{ad} \leq \nabla \leq \nabla_{rad}$).

However, a major issue in connection with convective cores is the question whether there is any mixing beyond the critical radius given by the Schwarzschild criterion (Shaviv

& Salpeter 1973) – as we say here, enhanced mixing. Naively thinking, we should expect rising convective elements, which approach the critical radius with a finite momentum, to shoot over by some length l_{ov} . That led to the popular name ‘overshooting’. However, other effects could lead to such enhanced mixing and the resultant core enlargement as well, and overshooting itself may actually only partly be involved. A competing process is, for example, rotationally induced meridional mixing (see, e.g., Flieger & Langer 1995). Furthermore, moderately different opacities might at least in principle also result in larger cores. That may appear to be a marginal problem at first glance, but – as we will show in the next section – the extent of the mixing core during hydrogen burning has tremendous consequences for the whole stellar evolution, which even go beyond the interest of stellar astrophysics, i.e. to galactic astrophysics.

Since any distinction between different sources of enhanced mixing beyond the convective cores is difficult to assess empirically, we have used a single, simple ‘ δ_{ov} prescription’. Its single parameter (see Section 3 for details) is intended to accumulate *all* effects which contribute to an enhanced mixing, and determines the overshooting length l_{ov} by which that mixing (through overshooting *and* other effects) exceeds and thus enlarges the classical convective core.

Motivated by the statement cited at the beginning, this work compares real late-type giants with known masses to matching evolutionary models in order to learn from the evolved stars about their internal properties during their past central hydrogen burning stages. We thus can specify δ_{ov} in particular, which is required for precise evolutionary tracks and their quantitatively reliable application in other fields such as galactic astrophysics. We use a well-tested, quick evolutionary code (see Eggleton 1971, 1972 and Pols et al. 1995; the latter is referred to as Paper I), which will be described in some detail in Section 3.

The study of galactic clusters has always been a crucial test to stellar evolution. The slope and redward extent of the upper main-sequence (MS) part of the cluster’s isochrone is somewhat sensitive to the amount of core-overshooting and other sources of enhanced mixing beyond the convective stellar cores. The most thorough test so far by such a study of clusters (Meynet, Mermilliod & Maeder 1993) yields evidence for a moderate amount of enhanced mixing (or overshooting) effects. However, any isochrone fit and the turn-off mass already depend on the choice of other model parameters. The accuracy is also limited by other uncertainties, e.g. by the often ill-defined location of the turn-off point.

As far as the giants in a cluster are concerned, their masses cannot be measured independently but are obtainable only by way of modelling the respective turn-off mass. Furthermore, only a few giants are massive enough to be unambiguously located in the blue loop (prominent blue loops require a mass in the range of 4 to 8 M_{\odot}) – of which α Persei is a well-studied example.

Eclipsing binaries and those few binaries with known inclination (obtained from precise speckle orbits) are the only sources of direct mass measurements, obtained from high-quality radial velocity curves, recorded for (ideally) both components. Systems with suitable giants, however,

are rare because they are necessarily restricted to wide orbits. For that reason, their radial velocity amplitudes are small and very precise measurements are required over a long time (periods range from years to decades). More details are given in Section 4. Since the binary nature considered for this study is intended to serve in a quantitative analysis of the evolution of genuine *single* stars, it is vital to select those few systems that are sufficiently wide that any possibility of interaction between the two components can be excluded, for any past stage.

We present case studies for selected crucial systems with giant primaries, using the best available data – partly based on unpublished results kindly provided by R. F. Griffin (see Section 5). All the eclipsing binaries studied here belong to the class of ζ Aurigae systems, which are well known for their ‘chromospheric eclipses’ and which provide unique chromospheric density profiles with direct spatial resolution (e.g. Wilson 1960). In addition, we have studied a few non-eclipsing systems, for which inclination and distance are already well determined from high-quality speckle orbits.

2 THE EFFECTS OF ‘OVERSHOOTING’ AND OTHER ENHANCED MIXING PROCESSES

Maeder (1975, 1976) was the first to discuss in detail the problem of overshooting and its effects on stellar evolution. For illustration, Fig. 1 compares the evolution of a standard (non-overshooting) 4- M_{\odot} , solar composition model with one that includes a moderate amount of enhanced mixing, reaching $l_{ov} \sim 0.27 H_p$ (pressure scaleheights) beyond the convective boundary of the core – computations are performed with our evolutionary code as described in the following section. Fig. 2 then compares the corresponding evolutionary tracks in a theoretical Hertzsprung–Russell (HR) diagram. It shows how the effects of enhanced mixing during central hydrogen burning become progressively magnified in the later evolutionary phases. It is for that reason that any empirical approach to calibrating the adjustable parameter connected to l_{ov} should be more sensitive if it can employ such evolved stars, i.e., late-type giants and supergiants.

In principle, all modern evolutionary codes agree about the following consequences of enhanced mixing of the convective core, all being obvious from Figs 1 and 2 (all other parameters are kept fixed).

- (i) Enhanced mixing prolongs the lifetime of core H burning (time spent on the MS) by feeding more H-rich material into the core than does standard convection alone.
- (ii) It widens the MS towards lower effective temperatures.
- (iii) The access to a larger hydrogen reservoir during core H burning enhances the He core mass left behind and thereby strongly changes the global characteristics of the following giant stages.
- (iv) As a side-effect, it significantly shortens the time spent during the initial phase of shell H burning, before the star crosses the Hertzsprung gap.
- (v) The luminosity is significantly enhanced on the later evolutionary track, in particular during the long-lived He-

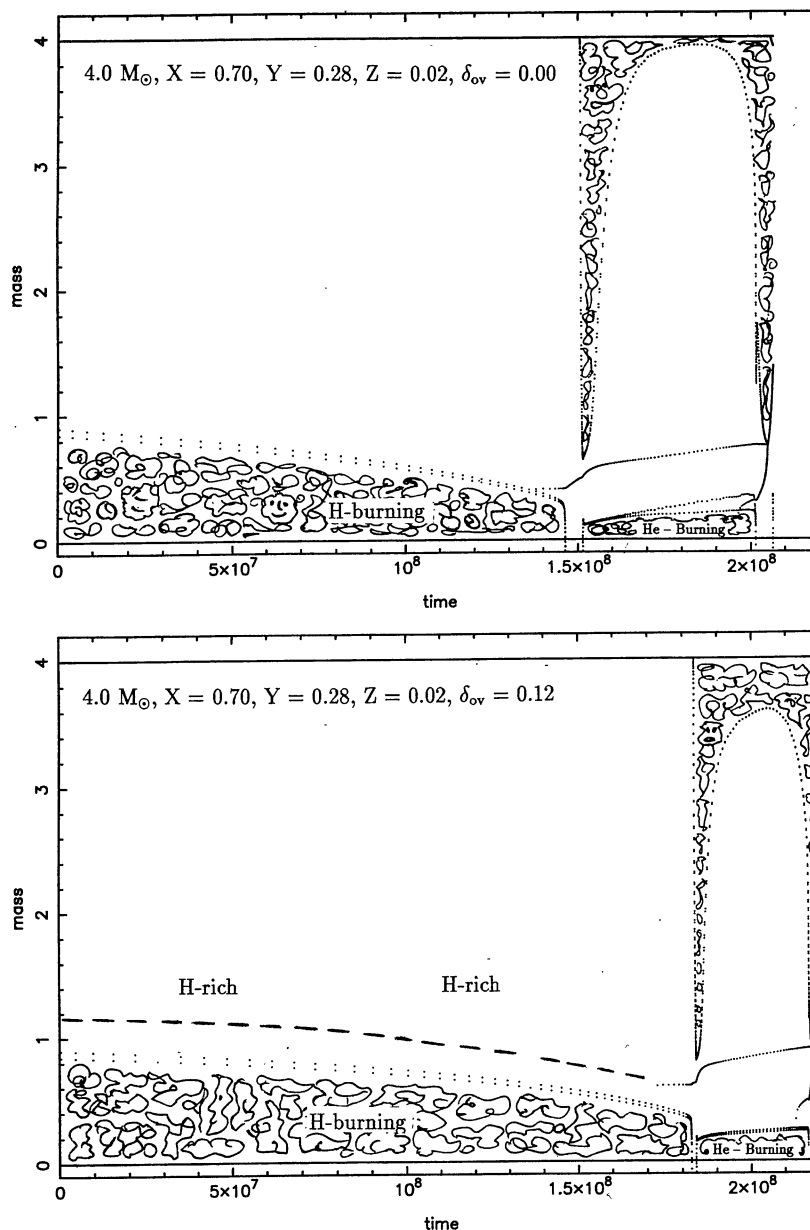


Figure 1. Comparison of $4.0\text{-}M_{\odot}$ models evolving without and with overshooting. Regions with convection (.....), slight convection and/or semiconvection (----) and overshooting (-.-) are indicated. The inner edge of the H- and He-burning shell are also marked. Note the different ratio of H/He core-burning lifetimes.

burning or ‘blue loop’ phase (we use the term ‘blue loop’ in a more generalized sense).

(vi) The life-time of core He burning (the time spent in the blue loop) is significantly shortened by the more intensive He consumption.

(vii) The changed abundance profile reduces the extent of the blue loop, which is a significant effect for the prominent blue loops of stars with $M_{*} \geq 4 M_{\odot}$.

In other words, the luminosity scale of the brighter giants ($M_{*} > 1.5 M_{\odot}$) is significantly changed and so are the evolutionary time-scales. Hence, enhanced mixing is of paramount importance to the stellar statistic in galactic astrophysics – in particular, the ratio of giants to MS stars

for both absolute and apparent numbers. Quantitatively, neglect of that effect could well amount to an underestimate of M_{ν} by one stellar magnitude for a given He-burning giant and to an overestimate of the ratio of giants to MS stars (absolute numbers) by a factor of 2! Therefore, a proper empirical calibration of the amount of enhanced mixing (which we shall hereafter refer to as ‘overshooting’) is of great importance, especially with respect to global evolutionary aspects and their quantitative application.

3 THE STELLAR EVOLUTION CODE

Our evolutionary code was designed to be robust and simple, and there are no complications with thin shell

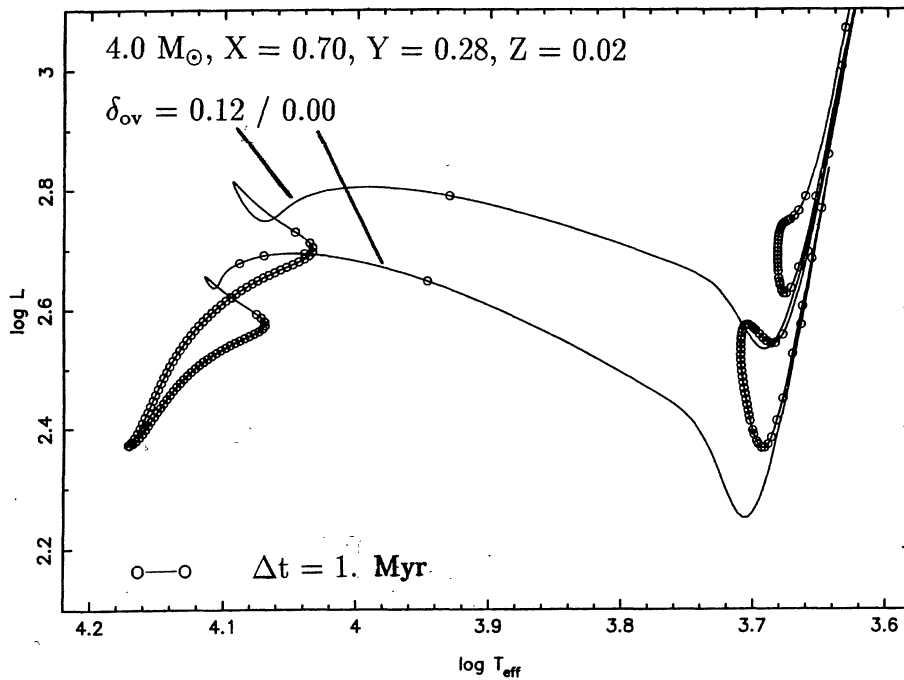


Figure 2. Evolutionary tracks of $4.0\text{-}M_{\odot}$ models without (lower track) and with (upper track) overshooting. The luminosity enhancement is most significant over the blue loop.

sources. The main intention of the code is to quantify the dominant features of stellar evolution as well as possible, but to be computationally efficient enough to be applied to more complex problems – originally in the evolution of close binary systems. That philosophy is also of great advantage for all those applications which need to cover stellar evolution in a *global* and, at the same time, *quantitatively* reliable way – e.g. in galactic astrophysics. A recent account of the updates of the code is given by Paper I, and a more detailed description, together with model grids for various metallicities, with and without overshooting, is in preparation (Pols, Schröder & Eggleton, Paper IV, in preparation). The main features of the code can be characterized here.

- (i) It uses a self-adapting mesh; structure and composition are solved simultaneously (Eggleton 1971, 1973).
- (ii) Convective mixing and semiconvection are treated as a diffusion process with a diffusion constant adopted as a function of $\nabla_{\text{rad}} - \nabla_{\text{ad}}$, while standard mixing-length theory is used to describe the heat transport (Eggleton 1972).
- (iii) Up-to-date opacities have been incorporated, i.e., OPAL (see Rogers & Iglesias 1992), and are complemented by those from Alexander & Ferguson (1994) at lower temperatures, i.e., $T \leq 10^4$ K.
- (iv) Nuclear rates, neutrino losses and the equation of state have also been updated recently (see Paper I and references therein).

A solar model compared by our code matches the Sun with a reasonable initial helium abundance of $Y=0.2812$ ($X=0.70$, $Z=0.0188$) and a mixing length of $l=2.0 H_p$ is required (see Paper I). He diffusion by gravitational settling is not considered. The luminosity scale of the models is shifted by the choice of initial helium abundance, which in

the case of the Sun can be assessed by helioseismology. Detailed solar models, which match observed helioseismological properties, require less He abundance in the outer layers of the Sun than most stellar (and our) models suggest and start with a uniform Y of about 0.25 (Kosovichev 1993). In that case, He diffusion by gravitational settling is included, which enhances the helium abundance in the core region, where it matters for the luminosity. Thus, He diffusion accounts for just the discrepancy of about 0.04 to our Y value (Christensen-Dalsgaard, Praffitt & Thompson 1993). We may therefore conclude that the luminosity scale of our models is genuinely correct, i.e. we do not need an artificial adjustment of Y .

Furthermore, we do not parametrize our model of overshooting by an overshooting length which is a fixed fraction of H_p (the ' H_p prescription'). Instead, we make an approach based on the stability criterion itself, the ' δ_{ov} prescription', by incorporating a condition that mixing occurs in a region with $\nabla_r > \nabla_a - \delta$. We define δ as the product of a specified constant δ_{ov} , our overshooting parameter, and a conveniently chosen factor which depends only on the ratio ζ of radiation pressure to gas pressure: $\delta = \delta_{\text{ov}} / (2.5 + 20\zeta + 16\zeta^2)$.

Thus our physical assumption is roughly that mixing occurs in regions that would classically be stable, provided that they are stable only by a suitably small margin δ . This avoids two unsatisfactory consequences of the H_p prescription: (a) since $H_p \rightarrow \infty$ at the centre, a triflingly small core would generate an enormous additional mixing region; (b) once the classical convective core shrinks to zero, the overshoot region abruptly shrinks to zero also. The δ prescription on the other hand gives a smooth decrease in the size of the overshoot region as $\nabla_r - \nabla_a$ at the centre decreases through zero, and continues on down to $-\delta$.

Our best-matching δ_{ov} value appears to be 0.12 and leads to overshooting lengths l_{ov} between 0.25 and $0.32 H_p$ for the mass range of 2.5 to $7 M_\odot$ (see Section 5). Typical δ values then range from 0.05 to 0.04, respectively.

4 ζ Aur-TYPE BINARY STARS

4.1 Eclipsing binary stars used so far

The potentially most powerful approach to the width of the MS makes use of the nearby eclipsing binaries. Eclipse photometry, radial velocity orbits and comparison of data from high-resolution spectroscopy with model atmospheres yield the best determined physical properties of the MS stars in those systems, i.e., radius R and T_{eff} (and hence L) and, most importantly, precise measurements of the masses M_* . Furthermore, both components of a binary not only need to fit on their respective evolutionary tracks, they also have to fit on the same isochrone as well (same age).

An extensive study, covering 45 binaries, was carried out by Andersen (1991). Andersen came to the conclusion that the best fits are obtained with a ‘mild’ amount of overshooting. But he pointed out that more precise results were difficult to achieve owing to a possible individual spread of metallicities, through which slightly different models all fit a given binary. Any fit also depends somewhat on the choice of opacities, which have been further improved in the meantime. For both these aspects, we refer to Pols et al. (in preparation, hereafter referred to as Paper III). We expect that more work will be devoted to this field, i.e., to more detailed spectroscopy and model atmospheres to derive the metallicities directly and thus to produce more stringent evidence.

In Andersen’s sample there are no eclipsing binaries which contain a bright giant. That is neither by accident nor on purpose, but is simply because of three unfavourable circumstances: the much smaller fraction of life-time spent in the giant stages; the inevitable exclusion of short-period systems (with more easily measurable radial velocity amplitudes) which at some point turn into semi-detached binaries once their evolved primaries have increased their radii by a critical amount; and the large ratio of radii of the two components, which means that the light curve alone contains less information on the orbital inclination and hence the radii.

In Paper III we re-examine the evidence for core overshooting from the Andersen sample by application of our evolutionary code. For most of those binaries, no firm conclusion can be drawn from an individual object alone. Those few somewhat evolved primaries could match an advanced MS model with overshooting, as well as a post-MS, shell H-burning non-overshooting model and mostly are – within the errors – on the same isochrone as their secondaries for both such cases. However, in one or two cases overshooting models do give a better fit, and from the combined data set a moderate amount of overshooting appears to be the more likely choice.

By contrast, we will demonstrate that for systems with a significantly evolved primary, i.e., in its core He-burning stages, any ambiguities are strongly reduced because of the magnified nature of the overshooting effects.

4.2 The special advantage of ζ Aur stars

The ζ Aur systems are rare cases of very wide eclipsing binaries, in which the primary is a late-type bright giant or supergiant. Their orbital separation equals typically 5 to 50 primary radii, and their periods range from 0.5 to 20 yr. The early-type companion usually dominates the ultraviolet (UV) part of the spectrum, while the much cooler but larger primary dominates from the visible into the IR. For some 60 years now, ζ Aur itself and two very similar binaries have been well known for the unique insight they offer into chromospheric structure. Shortly before and after eclipse the secondary behaves as a probing light source from behind (the chromospheric eclipse), and a phase-dependent chromospheric absorption line spectrum yields the density distribution of the observable ions with a sort of direct height resolution (e.g., Wilson 1960, Wright 1970 and Griffin et al. 1990).

In the past 12 years, several other spectroscopic binaries have turned out to belong to that exclusive family and several G giants have been added to this sample of primaries – see Schröder (1990) for a comparative study of those giants on the basis of the data available at that time, and more references therein. Those G giants turn out to be in their comparatively long-lived phase of core helium burning, to which we here, in a more generalizing manner, often refer as the ‘blue loop’. The best example is HR 6902 (V2291 Oph), which also happens to be the ζ Aur system with the most accurately known masses (Griffin & Griffin 1986). Table 1 gives the basic parameters as adopted here for the binaries with a measured mass ratio (keeping within the error bars of the empirical values). That selection also already shows the whole variety of primaries available from ζ Aur systems and crucial non-eclipsing binaries.

Except for the most favourable case of HR 6902, the measured mass ratios are somewhat uncertain, but the mass function is very well determined in all cases. See Griffin et al. (1990) and Bennett et al. (1997) for the most up-to-date system parameters of ζ Aur, Griffin et al. (1992) for τ Per.

There are a few nearby binaries with at least one giant component, for which a precise speckle orbit yields the inclination i of the system in the absence of any eclipses. We here discuss δ Sge, α Aur and η And. Data for δ Sge have been kindly provided by R. F. Griffin & R. E. M. Griffin (private communication, prior to publication). For α Aur, the precise masses and other system parameters from Barlow, Fekel & Scarfe (1993) have been adopted. For η And, the speckle orbit of Hummel et al. (1993) has been com-

Table 1. Observed masses of binaries with directly measured mass ratio q and well-determined inclination, including some ζ Aur systems and a few crucial non-eclipsing binaries.

object	spectral type	q	M_{prim}/M_\odot	M_{sec}/M_\odot
HR6902	G9IIb+B8.5V	1.31 ± 0.02	3.86 ± 0.15	2.95 ± 0.09
ζ Aur	K4Iab+B6V	1.24 ± 0.06	6.3 ± 0.7	5.1 ± 0.4
τ Per	G8IIIa+A2V	1.30 ± 0.15	2.4 ± 0.6	1.8 ± 0.4
δ Sge*	M2Ib-II+B9V	1.27 ± 0.06	3.4 ± 0.7	2.7 ± 0.5
α Aur*	G8III+G0III	1.05 ± 0.02	2.61 ± 0.08	2.49 ± 0.07
η And*	2x G8 III-II	1.06 ± 0.02	2.39 ± 0.14	2.26 ± 0.14

*No eclipses; inclination determined by speckle orbit.

binned with the individual spectroscopic $m \sin^3(i)$ values from Griffin (private communication).

To make best use of those less well-determined cases, e.g., τ Per with a less well-determined mass ratio and 31 Cyg (V695 Cyg), 32 Cyg (V1488 Cyg) and HR 2554 (V415 Car) without any direct measurements of that quantity, we need to explore the whole possible range of mass ratios for each system. Any such range yields a well-defined family of possible mass pairs, i.e. those which exactly satisfy the mass function. From those we have adopted those pairs of masses for which the track of the secondary fits best in L and T_{eff} (see also Tables 1 and 2). We may do so, since the luminosity scale of the theoretical zero-age main-sequence (ZAMS) obtained with our evolutionary code is in good agreement with the Andersen (1991) star sample, the precise mass values of which provide a stringent M_* - L relation test on the MS (see Paper III).

When compared with mass values of eclipsing binaries with two MS stars, most mass values measured from ζ Aur systems have a lower precision. That reflects the observational problems with measuring much lower radial velocity amplitudes from much more complex composite spectra. The same is true for deriving precise effective temperatures. Nevertheless, the advantages of fitting theoretical models to ζ Aur binaries are summarized as follows.

(i) ζ Aur binaries provide well-known, complete sets of system parameters – including radii directly measured from their eclipse light curves.

(ii) They also include the most *precisely measured giant masses* ($\sin i \simeq 1$ owing to their eclipsing nature). There are only a few non-eclipsing nearby wide binaries with an equally evolved primary, which yield the same quality of information, because speckle orbits provide their $\sin i$ value and geometrical parallaxes provide their absolute dimensions and luminosities.

(iii) ζ Aur binaries are wide enough – but for one possible exception – to ensure a separate evolution of their components. Hence there are no complex effects of binary-related interaction.

Table 2. Adopted physical parameters of ζ Aur and related binaries.

star	T_{eff}	R_*/R_{\odot}	$\log L/L_{\odot}$	M/M_{\odot}
ζ Aur giant	4150 K	151	3.78	6.6
ζ Aur B star	15000 K	5.1	3.07	5.2
32 Cyg giant	4100 K	170	3.85	7.2
32 Cyg B star	14000 K	3.1	2.51	4.1
31 Cyg giant	4100 K	170	3.85	7.2
31 Cyg B star	16500 K	4.0	3.03	5.5
HR 6902 giant	4850 K	32.9	2.73	4.0
HR 6902 B star	11000 K	3.0	2.07	3.0
τ Per giant	5150 K	15.8	2.20	2.8
τ Per A star	9300 K	2.2	1.45	2.0
HR 2554 giant	5050 K	31.1	2.75	3.7
HR 2554 A star	9300 K	2.0	1.44	2.3
δ Sge giant	3600 K	157	3.57	3.4
δ Sge B star	9900 K	3.3	1.97	2.7
α Aur A	5070 K	11.4	1.87	2.6
α Aur B	5900 K	8.8	1.85	2.5
η And A	5050 K	10.5	1.81	2.4
η And B	5000 K	8.5	1.60	2.25

(iv) Some of the ζ Aur giants are in their blue loop phase and can be used to probe the core overshooting with the highest possible sensitivity (see Section 2).

(v) Both components have the same age: the exact position of the MS secondary on its track provides an additional check of a fitting parameter set and significantly reduces any ambiguities.

In this study we focus on those special advantages. The most significant results are presented in the following section.

5 EVOLUTIONARY ANALYSIS OF INDIVIDUAL CASES

Finding out which parameter set – and, in particular, how much overshooting – provides the best match of a well-known giant and its secondary (for the same age) is a sort of calibration process. We may even speak of calibrated evolutionary tracks, because some of their parameters have been adjusted to reproduce empirical data optimally. That calibration is of course only valid for the range of masses discussed here and its accuracy depends on the quality of the binary data available.

With the multitude of parameters, it requires some thought to find the best matching model – despite the favourable circumstances listed above. Since the structure of the observational uncertainties is different from system to system, merely representable by error bars, we have settled on a non-automatic-fit strategy, which gives larger weight to the slower evolutionary phases – reflecting a larger likelihood of the star being found on this particular part of the evolutionary track. We also tried to minimize any differences in the parameter sets of different systems. For example, no individual adjustments of the chemical composition in both Y and Z were made but those parameters were assumed to be coupled as $\Delta Y/\Delta Z = 2.0$ in the few cases in which deviations from a quasi-solar composition have to be considered. Furthermore, we kept the mixing length fixed at $\alpha = 2.0$ as derived from our solar model. In fact, as we show below, there is no evidence for any dependence of α on gravity, at least not within the observational uncertainties.

5.1 HR 6902

Because of its precisely known masses, HR 6902 (V2291 Oph) is the most crucial system among those which contain a bright G giant. It should therefore give a very tight test of the overshooting issue. A detailed discussion of the absolute luminosities and effective temperatures of the components can be found in Griffin et al. (1995) and Schröder, Marshall & Griffin (1996). We adopt the values of the latter paper – i.e., $R = 32.9 \pm 1.5 R_{\odot}$, $\log L = 2.73 \pm 0.08$, $T_{\text{eff}} = 4850 \text{ K} \pm 100 \text{ K}$ for the giant primary and $R = 3.0 \pm 0.2 R_{\odot}$, $\log L = 2.07 \pm 0.08$, $T_{\text{eff}} = 11000 \text{ K} \pm 500 \text{ K}$ for the secondary (see also Table 2). Fig. 3(a) shows a comparison of the positions of the components in a theoretical HR diagram with the respective best-fitting evolutionary tracks, and Fig. 3(b) compares the observed $\log L$ and $\log T_{\text{eff}}$ with an isochrone computed with the same parameters as the tracks. Quasi-solar abundances have been used as indicated and the overshooting length l_{ov}

of the convective core in the young MS $4-M_{\odot}$ model needed to be $\sim 0.27 H_p$ ($\delta_{ov}=0.12$).

The giant is not cool enough to fit on the theoretical giant branch (GB) or asymptotic giant branch (AGB), without an otherwise unjustified change in the mixing length. The tracks as shown here provide a good fit for quasi-solar abundances and for the same mixing length of $2.0 H_p$. They also provide the best probability, as most of the lifetime of the giant is spent in its blue loop where this track puts it (Fig.

3a). Since in any case such a giant evolves more quickly than its MS secondary, it is the giant which defines the age of the system (depending only on its exact mass). Note that the respective age mark on the track of the slightly evolved MS secondary fits the position of the latter very well, too.

The required amount of core overshooting, $\delta_{ov}=0.12$, leading to $l_{ov} \simeq 0.27 H_p$, is well constrained: less overshooting puts the corresponding blue loop at too low a luminosity (see Fig. 4). Without any overshooting, blue loop

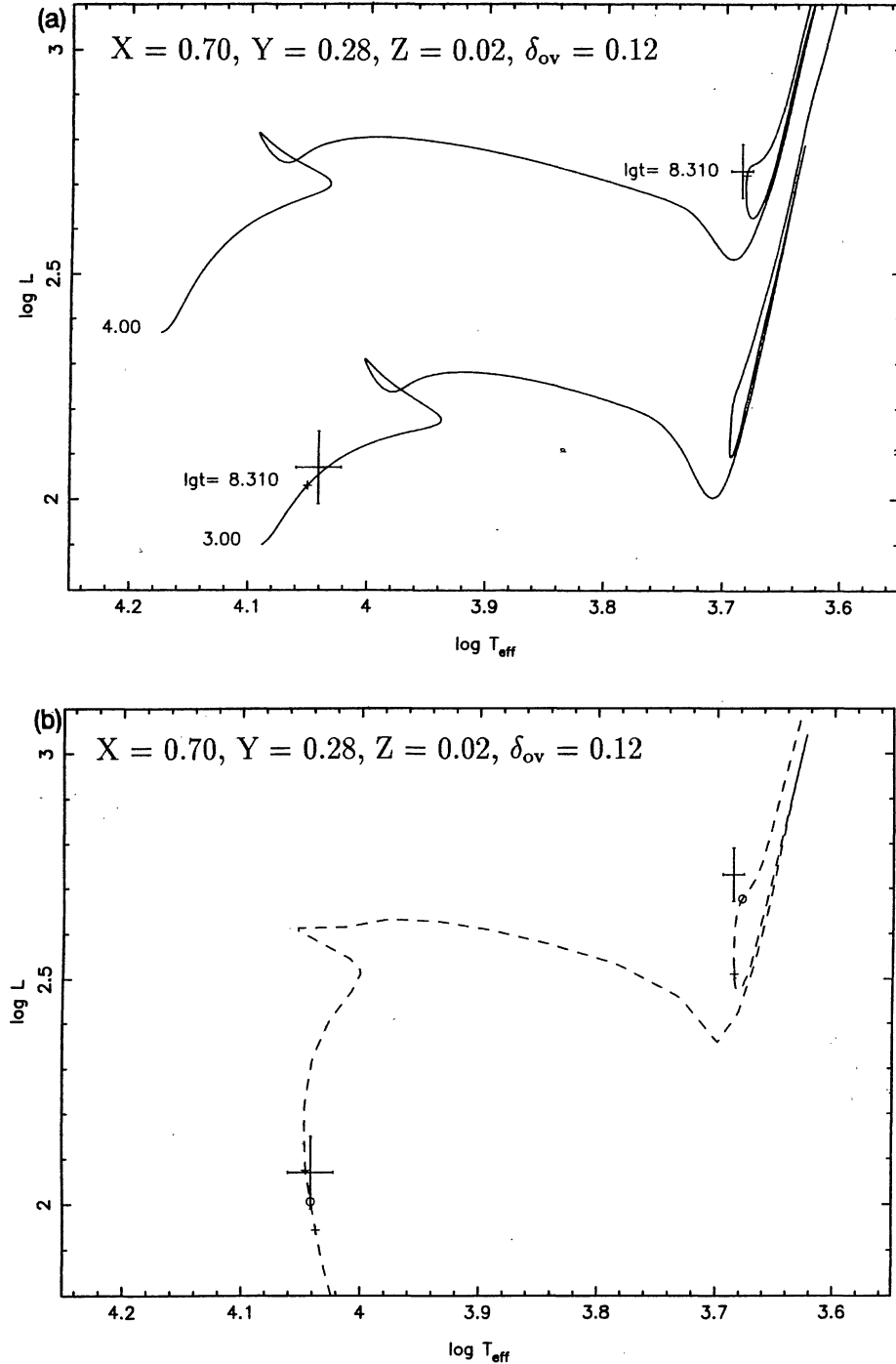


Figure 3. HR 6902: comparison of observational data and (a) above, evolutionary tracks for quoted masses (age indicated by +) in a theoretical HR diagram and (b) below, of an isochrone (mass indicated by o) for an age of 230 Myr.

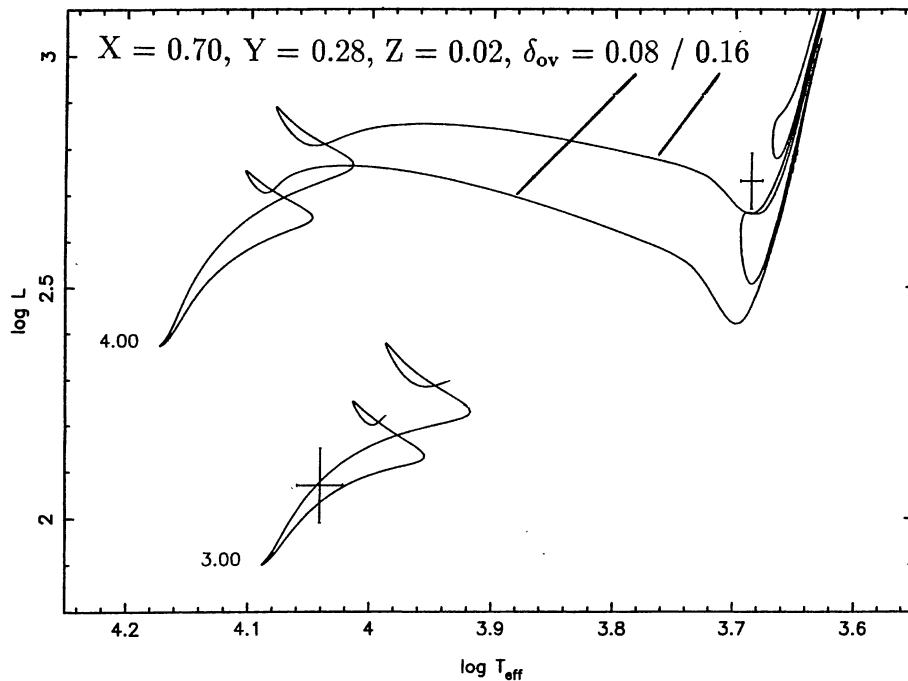


Figure 4. HR 6902: comparison of observational data and evolutionary tracks in a theoretical HR diagram for 30 per cent less and 30 per cent more overshooting than in Fig. 3.

luminosities are approximately one magnitude less, which is roughly four times the error bar in L (giant). A larger δ_{ov} not only causes the blue loop luminosity to rise further – a moderate rise is not excluded by the HR diagram position of the giant – but the changed hydrogen profile, as left behind by the core hydrogen burning, then also suppresses the blue loop more noticeably. That effect was already pointed out by Weigert (1975). The effective temperature of the giant, which stems directly from a detailed non-local thermodynamic equilibrium (non-LTE) analysis (Marshall 1994, 1996) is then too high to be matched by a track without an ad hoc change of α , the mixing length parameter. Models for higher than solar metallicities (for which there may be some evidence, see Marshall 1994, 1996) match even worse. Fig. 4 also demonstrates the superior sensitivity to overshooting of the blue loop in comparison to the MS stages, which are quite tolerant (within the error bars) to a change in about 30 per cent of the amount of overshoot.

We may therefore say that this system calibrates $\delta_{ov} = 0.12$ to within approximately 30 per cent. We find that the corresponding l_{ov} value is $0.27 H_p$ (within about $0.08 H_p$). In the following we test the validity of that δ_{ov} value, along with the other parameters (α , abundances X, Y, Z), for some range of masses by analysing the other ζ Aur systems listed in Table 2.

5.2 ζ Aurigae

For tracks with quasi-solar abundances ($Z = 0.02, Y = 0.28$), as shown in Fig. 5(a), the K supergiant of ζ Aur is cool enough to be matched by a theoretical GB or AGB. In terms of likelihood, the slower passages down from the GB and up from the foot of the AGB are to be pre-

ferred. The former alternative gives a bad, the latter a marginal, match for the secondary on its track, when the common age is considered (for which the models give 55 and 62 Myr, respectively). For the pair of masses which yields both matching luminosity and a matching luminosity difference, a mass ratio of $q = M_K/M_B = 1.27$ is required. This is well within the measured value of 1.3 ± 0.1 according to Griffin et al. (1990) and not much different from the 1.21 ± 0.03 of Bennett et al. (1997). The former paper also gives and cites further observational properties (photometry and spectroscopy) from which we have deduced the physical quantities of the ζ Aur components as stated in Table 2.

Since that fit is not implausible but also not as good as for HR 6902, we tried alternative tracks for a slightly higher metallicity ($Z = 0.03$ and $Y = 0.30$ according to our choice of $\Delta Y/\Delta Z = 2.0$), as shown in Fig. 5(b). The best fit, compromising for the luminosities and their difference, is obtained for a mass pair requiring $q = 1.30$. With this higher metallicity, the blue loop is strongly suppressed. That model predicts that the K giant is still well within its central He-burning phase, and the fit of the secondary to its track at the corresponding age is again marginal.

The best fit (but not much better) is obtained somewhere in between those two alternatives, with $Z \approx 0.025$ and the primary being on the point of leaving its blue loop. The uncertainties in the observed quantities, however, allow the full range between the two alternatives shown in Fig. 5. In any case, we can give a low probability to any core overshooting resulting in l_{ov} much less than $0.31 H_p$ (i.e., δ_{ov} much less than 0.12), since the K supergiant would then be higher up on its AGB, where it evolves much more quickly, and where it is therefore much less likely to be. For larger values of overshooting the blue loop is again strongly sup-

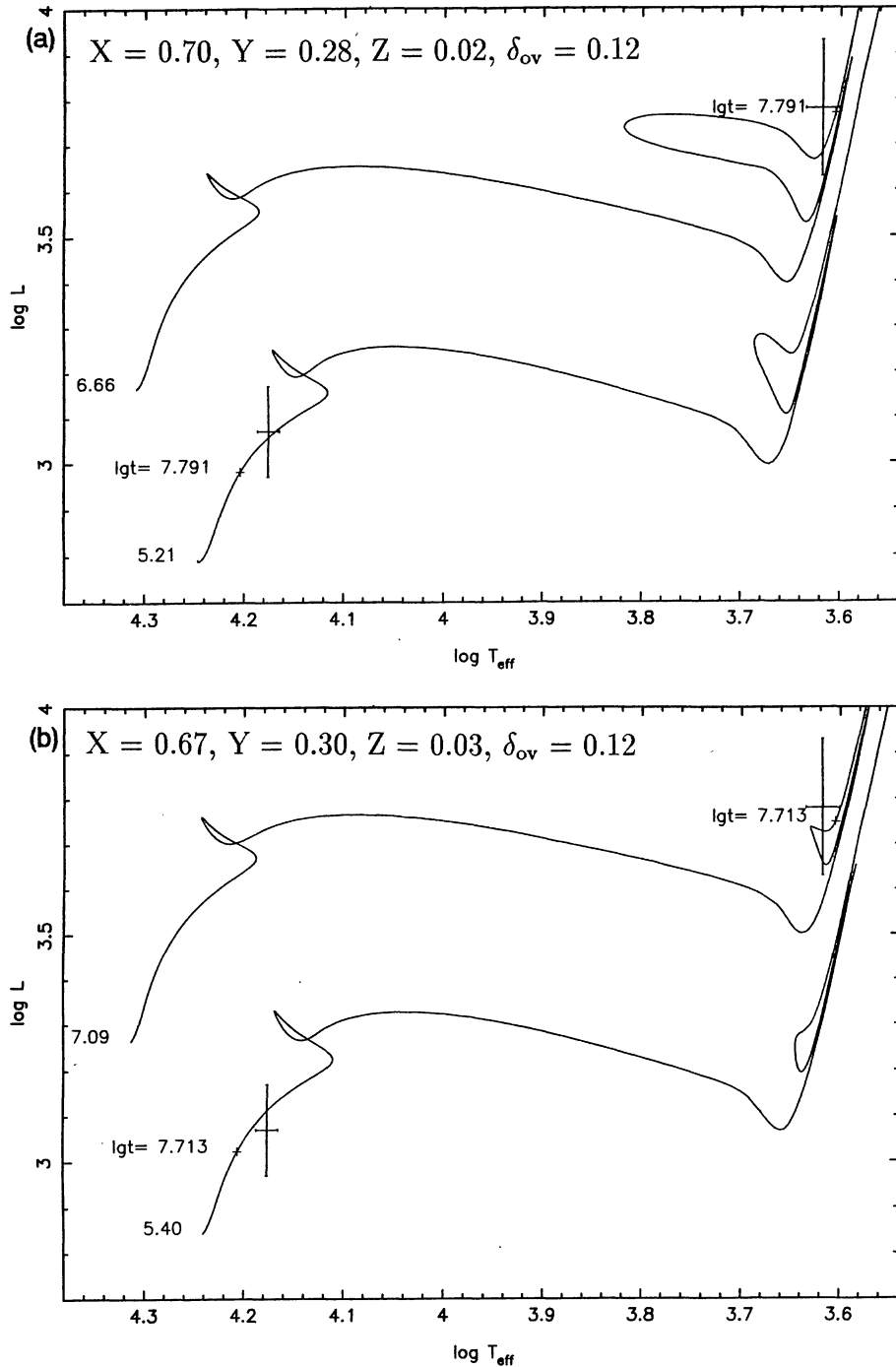


Figure 5. ζ Aur: comparison of observational data and matching evolutionary tracks for quasi-solar metallicity (a), above, and for enhanced metallicity (b), below.

pressed, which does not match a star like, e.g., α Per (see Section 6, below), a much earlier supergiant of similar mass. Altogether, a δ_{ov} of 0.12 is confirmed within ~ 30 per cent of its value.

5.3 τ Persei

τ Per may be an example of a lower-than-solar metallicity ($Z=0.01$ and $Y=0.26$ accordingly), for which there is also

some spectroscopic evidence in the secondary (Griffin et al. 1992, see also for all physical parameters). From the evolutionary point of view, the effective temperature of the G giant is too high in comparison with the maximum T_{eff} for any theoretical blue loop with quasi-solar abundances. Furthermore, the mass values required for the low-metallicity fit (Fig. 6) correspond to $q=1.4$ (implying masses of 2.8 and $2.0 M_{\odot}$), in fair agreement with the 1.3 ± 0.15 (implying masses of 2.4 and $1.85 M_{\odot}$) observed by Griffin et al. (1992).

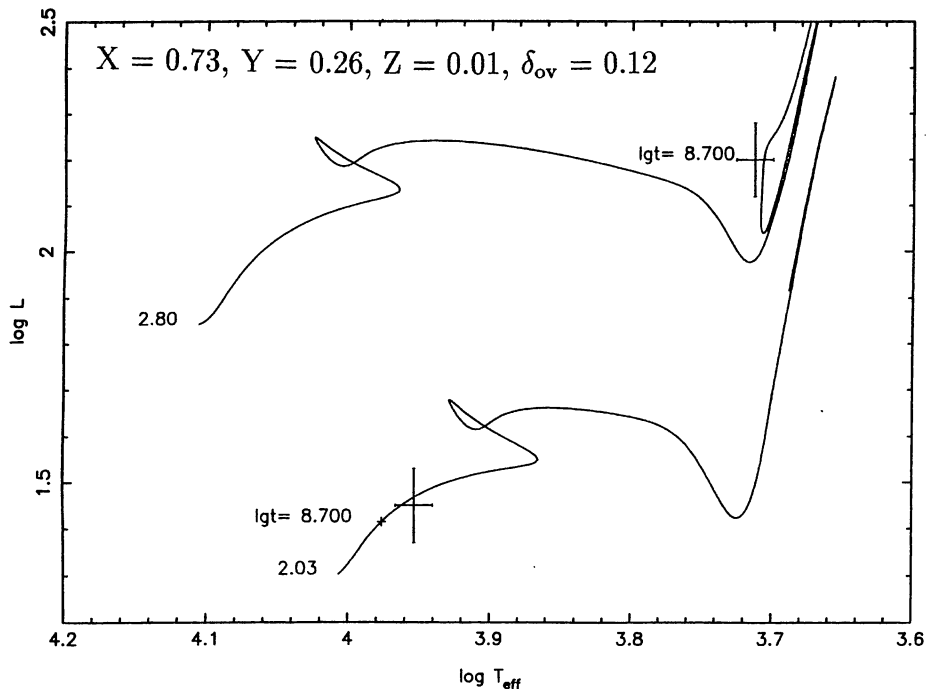


Figure 6. τ Per: comparison of observational data and matching evolutionary tracks for lower than solar metallicity.

By contrast, matching the luminosity of the MS companion with quasi-solar abundances requires a significantly larger deviation from that value – to even larger values.

Since the theoretical blue loop luminosity is in any case too small when computed without core-overshooting, the observational quantities of τ Per are in good agreement with the same moderate amount of overshooting as derived above from matching tracks for HR 6902.

5.4 Other ζ Aur systems of potential interest

As far as core overshooting is concerned, we have expanded this critical test of evolutionary theory and its parametrization to the other two known G giant primaries, HR 2554 and 22 Vul. However, for the former, which is a southern sky near-equivalent of HR 6902, no direct mass ratio is available and, in addition, the mass function may be less certain. The most recent orbital elements for HR 2554 were published by Wilson & Huffer (1918) (see Schröder & Hünisch 1992 for a recent discussion of the HR 2554 system parameters). We therefore do not obtain strong additional constraints from this system. Nevertheless, a secondary mass of $2.25 M_{\odot}$ is required for a track of appropriate luminosity, and the mass function (0.31) then implies a realistic primary mass of $3.7 M_{\odot}$ (see Table 2). For that, the theoretical blue loop catches up with the luminosity of the primary only if δ_{ov} is at least 0.12, or better 0.15, which correspond to $l_{ov} \sim 0.27\text{--}0.34 H_p$, respectively.

22 Vul is the system with the lowest ratio of orbital separation to giant radius – sufficiently low to have some possibility for a mass-exchange in the past, i.e. when the primary was near the top of its GB and might have filled its Roche lobe for some time. We only need to assume that the 22 Vul primary may then have started with a somewhat larger mass and thus reached larger radii on the GB than a track for its

present mass suggests. Furthermore, the well determined eccentricity is 0.00 (for a recent review of its system parameters, see Griffin et al. 1993). Those are two strong indicators that our initial assumption of an independent evolution is not justified for this system.

There is – not surprisingly, then – no plausible combination of masses (with respect to mass function and mass ratio) for which we can obtain a pair of tracks which match both components at the same time, the companion on the MS and the giant on its blue loop. The 22 Vul G giant has a luminosity much too large relative to its companion. Its effective temperature is also not low enough to be matched by a theoretical GB or AGB. We have therefore excluded this probably peculiar system from the discussion of parameter calibration.

The remaining two ζ Aur binaries are 31 Cyg and 32 Cyg, in which the primaries are very similar to the one in ζ Aur. As already mentioned, their mass ratios also remain to be measured. For 32 Cyg, however, it may soon be available as an off-shoot from observations with the *Hubble Space Telescope* presently in progress. Both systems can be fitted excellently with quasi-solar abundances and the same overshoot ($\delta_{ov}=0.12$) as derived from the other binaries; however neither system can provide strong constraints owing to the remaining uncertainty of its masses. Both K supergiants seem to be at the upper end of the blue loop, to have a mass of $7.2 M_{\odot}$ and an age of 52 Myr.

To derive the positions of their components in the HR diagram, we use eclipse models (both orbital velocities known) together with the B star properties [T_{eff} , Θ and $E(B-V)$] derived from *IUE* spectra. That combination yields radius R , distance d , bolometric correction BC and luminosity L of the B stars and thus implicitly of the giants as well. For the effective temperature and BC of the giants, we adopt 4100 K and -1.0 mag according to their spectral

type (slightly later than ζ Aur) and colours, by comparison to the colour table of the ATLAS9 Kurucz model atmospheres (see Kurucz 1991). Furthermore, the radial velocity orbits yield the mass functions, f_m , which constrain the possible pairs of masses to be unique for a given mass ratio. Wright (1970) gives $f_m = 1.01 M_\odot$ for 31 Cyg, $\sin i = 1.0$; and Griffin (private communication) gives $f_m = 0.474 M_\odot$ for 32 Cyg, $i = 78^\circ$ in grazing eclipse. We adopted those mass pairs for which evolutionary tracks match best on the luminosity scale. Our resulting sets of physical properties, thus made consistent with evolutionary tracks, are listed in Table 2.

5.5 Crucial non-eclipsing binaries: δ Sag, α Aur and η And

5.5.1 δ Sagittae

The M supergiant in δ Sge is so much more luminous and cooler than any likely theoretical blue loop, that it is clearly in the final stages of ascending the AGB (Fig. 7). In consequence, it is insensitive to any choice of l_{ov} , but it provides a good test of whether the choice of the mixing length for its outer convective regions is the same here, taken as $\alpha = l/H_p$, as in the case of the Sun ($\alpha = 2.0$). In fact, the resultant track falls within the error bars, as we show below.

The uncertainties in BC and T_{eff} are quite large for any such M-type giant – any thorough evaluation is a complex non-LTE and molecular-opacity problem. We have adopted 3600 K and $BC = -1.75$ mag, according to the spectral type and a plausible model for the colours of the system, which are $B - V = 1.39$ mag, $U - B = 0.94$ mag according to Hoffleit & Jaschek (1982), if $E(B - V) = 0.02$ mag: the system is composed of a B9.5 V secondary of $T_{\text{eff}} = 9900$ K as determined from a comparison of IUE spec-

tra with Kurucz model atmospheres (Kurucz 1979), and a typical M2 supergiant. Representing the former by colours $B - V = -0.04$ mag and $U - B = -0.09$ mag, $BC = -0.355$ mag, as suggested by the Kurucz model atmospheres for $\log g = 3.8$ (Kurucz 1991), we get a reasonable set of colours for the M2 supergiant of $B - V = 1.7$ mag and $U - B = 2.2$ mag.

A comparison of speckle orbital elements with spectroscopic orbital elements (R. F. Griffin & R. E. M. Griffin, private communication) yields a distance modulus $m - M = 6.3$ mag, and thus the absolute luminosities – see also Table 2.

We estimate the error bar on T_{eff} to be as much as 250 K, the one on BC to be 0.5 mag. With those uncertainties, we nevertheless may exclude values of α larger than about 2.3 and smaller than about 1.5, at least for quasi-solar metallicity. Since we are comparing two extreme cases of gravity, differing by four orders of magnitude, any dependence of the mixing length – taken in pressure scaleheights – on $\log g$ must hence be very subtle.

The masses adopted for the evolutionary tracks in Fig. 7 correspond exactly to the measured mass function $f_m = 0.135 M_\odot$, if $i = 39.5^\circ$ as in close agreement with a fit to the available speckle data, and if $q = 1.27$ as observed (again Griffin & Griffin, private communication).

5.5.2 α Aurigae

By combining intensive speckle and radial velocity measurements to a three-dimensional orbital solution, Barlow, Fekel & Scarfe (1993) give a set of refined physical parameters for this well-known system (Table 2). The secondary is already significantly evolved itself and on its fairly quick passage to the GB (see Fig. 8). Therefore, unlike the other secondaries, α Aur B is the most crucial component to

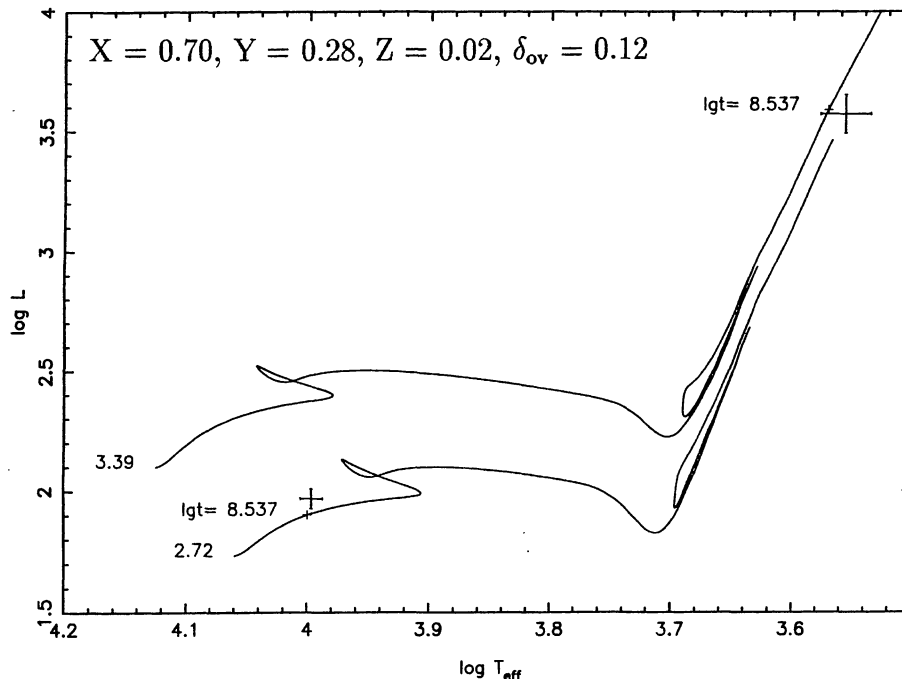


Figure 7. δ Sge: comparison of observational data and matching evolutionary tracks for quasi-solar metallicity.

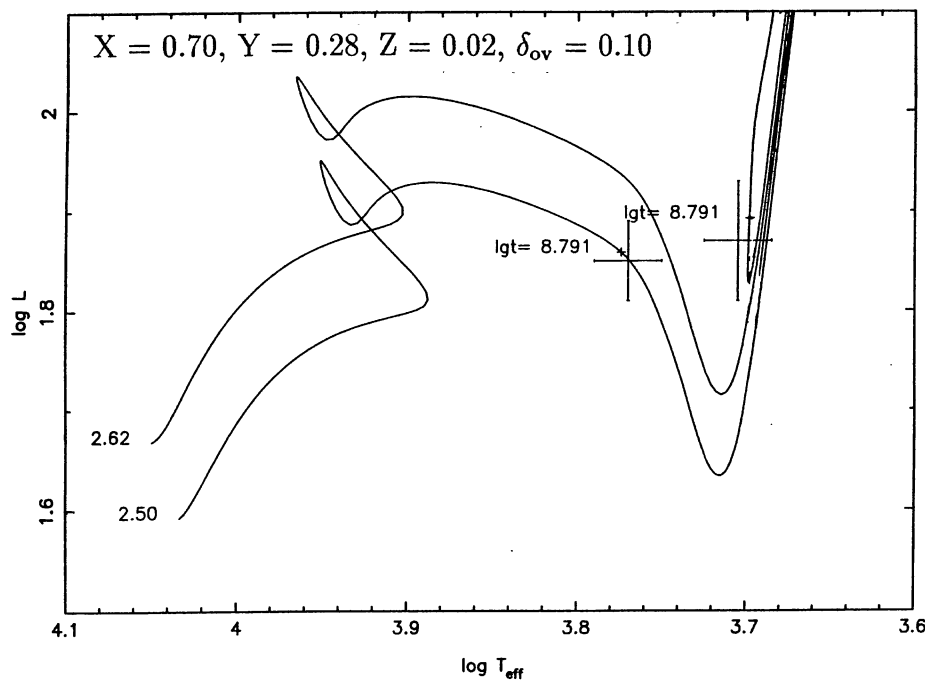


Figure 8. α Aur: comparison of observational data and matching evolutionary tracks for quasi-solar metallicity.

determine both the age of the binary (618 Myr from our best-matching model) and also the corresponding exact position of the primary on its blue loop.

The high quality of the empirical data, especially the small uncertainty in the masses ($0.08 M_{\odot}$, each), makes this system another very crucial test object for the amount of overshooting, at a significantly lower mass than HR 6902. We find that $\delta_{ov} = 0.10 \pm 0.02$ is required, corresponding to $l_{ov} = 0.20 H_p$.

5.5.3 η Andromedae

For η And there is an accurate speckle orbit available from Hummel et al. (1993), which we combined with the latest spectroscopic orbital solution of Griffin (private communication 1996) to determine the distance and absolute luminosities. Griffin's observations also yield accurate masses for both components individually (see Table 2). We estimate their accuracy to be about $0.15 M_{\odot}$ each. Approximately two-thirds of the mass uncertainty comes from the error bar on $\sin i$, one third from the radial velocity measurements.

As Fig. 9 demonstrates, the secondary here is already on its ascent to the GB and determines the age of the system (838 Myr from our model). In fact, this circumstance defines the location of the primary in its blue loop very well and we have another very accurate test of the amount of overshooting required. We derive a $\delta_{ov} = 0.105 \pm 0.02$ as an optimum fit, corresponding to $l_{ov} = 0.21 H_p$.

6 DISCUSSION

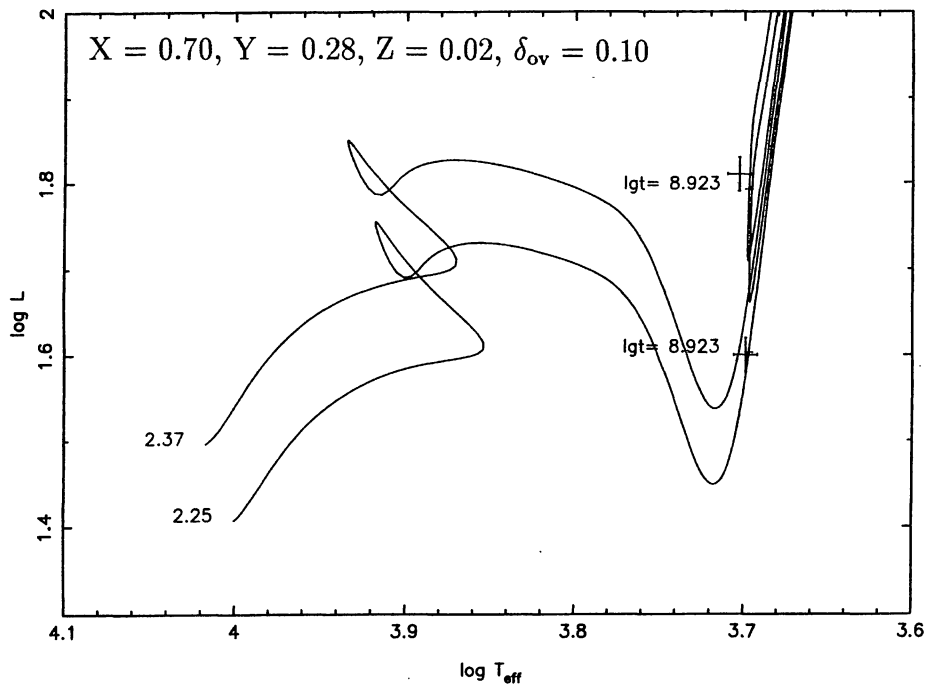
Within the observational error bars of M_* , L , T_{eff} , all well known ζ Aur systems can be matched by evolutionary tracks computed with our evolutionary code and a consistent

parameter set, including in particular $\alpha = 2.0$ (chosen by being the best choice for the Sun) and a fixed $\delta_{ov} = 0.12$ which leads to $l_{ov} \approx 0.24$ to $0.32 H_p$. Abundances are either quasi-solar with $Y = 0.28$, $Z = 0.02$, or – when differing from that – with $\Delta Y / \Delta Z = 2.0$. It is worth noting that the binary with the most precise observational data, HR 6902, also provides the best agreement with its evolutionary tracks.

Fig. 10 summarizes the results of Section 5, as far as the required extent of the enhanced mixing is concerned, represented by an overshooting length from the convective cores of young MS stars. The slight increase of l_{ov}/H_p , as invoked by our ‘ δ prescription’ and using $\delta_{ov} = 0.12$, seems to be real, and this fixed parameter can be used with our code for almost the entire mass range of 2.5 to $7 M_{\odot}$ to account for overshooting effects.

The absolute values of adjustable parameters such as the overshooting length – although derived from the empirical calibration of evolutionary tracks – may depend on the details of the input physics represented by a particular code. Hence, parameters may not compare strictly with their values derived from the calibration of other codes. Instead, it should be better to compare, directly, how well they can match certain crucial examples.

We therefore tried our set of parameters (including the same δ_{ov}) on the α Per cluster, for which the Geneva code gives a perfect fit (α Per is supposed to be at the blue tip of its blue loop, see fig. 4 of Meynet et al. 1993). To compare the resultant isochrone with the upper end of the cluster MS and α Per itself, a transformation from the theoretical HR diagram into the M_V versus $B - V$ plane is required. We used the Kurucz ATLAS9 (see Kurucz 1991) tables of $B - V$ and BC for given T_{eff} and $\log g$. Neither the F-type supergiant nor the hot MS B-type stars are near the extremes of this table, so its application should not introduce significant errors. See Fig. 11 for the resulting comparison.



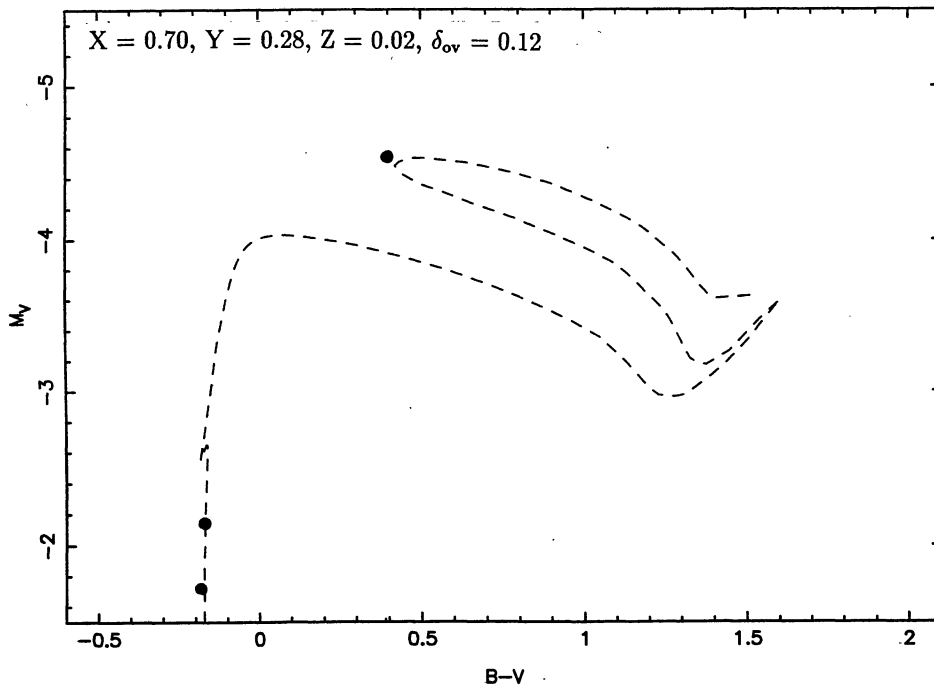


Figure 11. α Per and the turn-off-point from its cluster MS: comparison of observations with an isochrone computed for the same calibrated parameter set as derived from HR 6902 and other ζ Aur systems, transformed into the M_V versus $B - V$ plane.

the giant is thus much less likely to be found there. Again, the same situation occurs for the other binaries of which, by statistical arguments, a large majority should rather be found in the long-lived blue-loop phase. Therefore, it appears to be much more likely that α is indeed (almost) invariable, i.e., of the value 2.0 in the whole range between the extreme cases of the Sun ($\log g = 4.4$) and δ Sge ($\log g = 0.6$).

Unfortunately, we cannot contribute to the interesting debate of a strong decline or even breakdown of overshooting around a mass of $1.5 M_{\odot}$, as discussed by several authors from both empirical and theoretical points of view (e.g. Alongi et al. 1993, Meynet et al. 1993, Roxburgh 1992, Umezu 1995). Such masses are not accessible by the approach shown here, and only further work on appropriate clusters and seismological studies of stars like η Boo (Guenther & Demarque 1996) may clarify this problem.

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